

# Extended Spherical Collapse and the Accelerating Universe

A. Del Popolo\*

*Astronomy Department, University of Catania, Italy and  
Departamento de Astronomia, Universidade de São Paulo,  
Rua do Matão 1226, 05508-900, São Paulo, SP, Brazil*

F. Pace†

*Institute of Cosmology and Gravitation, University of Portsmouth,  
Dennis Sciama Building, Portsmouth, PO1 3FX, U.K.*

J. A. S. Lima‡

*Departamento de Astronomia, Universidade de São Paulo,  
Rua do Matão 1226, 05508-900, São Paulo, SP, Brazil  
(Dated: Received July 26, 2012; accepted ?)*

The influence of the shear stress and angular momentum on the nonlinear spherical collapse model is discussed in the framework of the Einstein-de Sitter (EdS) and  $\Lambda$ CDM models. By assuming that the vacuum component is not clustering within the homogeneous nonspherical overdensities, we show how the local rotation and shear affects the linear density threshold for collapse of the non-relativistic component ( $\delta_c$ ) and its virial overdensity ( $\Delta_v$ ). It is also found that the net effect of shear and rotation in galactic scale is responsible for higher values of the linear overdensity parameter as compared with the standard spherical collapse model (no shear and rotation).

PACS numbers:

## I. INTRODUCTION

Current analyses of high quality cosmological data are suggesting a cosmic expansion history involving some sort of dark energy and a flat spatial geometry in order to explain the recent accelerating expansion of the universe [1–3].

Among a number of possibilities to describe the dark energy (DE) component, the simplest one is by means of a cosmological constant  $\Lambda$  (see [4] for reviews), usually interpreted as the vacuum energy density ( $\rho_v$ ) which acts on the Friedmann's equations as a perfect fluid with negative pressure ( $p_v = -\rho_v$ ). In the present cosmic concordance  $\Lambda$ CDM model, the overall cosmic fluid contains non-relativistic matter (baryons + cold dark matter,  $\Omega_{nr} = 0.274$ ) plus a vacuum energy density ( $\Omega_\Lambda = 0.726$ ) that fits accurately the current observational data and thus it provides an excellent scenario to describe the present observed universe [5].

Nowadays, one of the most challenging problems in the so-called  $\Lambda$ CDM cosmology is to understand the role played by the different cosmic components during the non-linear regime of gravitational clustering and how the many possible physical effects contribute to determine the total mass of virialized halos (galaxy and galaxy clusters). A popular analytical approach to study the non-linear evolution of perturbations of dark matter (in the

presence of a non-clustered dark energy (DE) is the standard spherical collapse model (SSCM) proposed in the seminal paper of Gunn and Gott [6] and extended in subsequent papers [7]. The model describes how a spherical symmetric overdensity decouples from the Hubble flow, slows down, turns around and collapse.

In the last decade, the SSCM has been applied to study density perturbation evolution and structure formation in presence of DE. However, when solving the density contrast ( $\delta$ ) in the SSCM, the local shear ( $\sigma$ ) and rotation ( $\omega$ ) parameters are usually not taken into account. While the first assumption is correct, since for a sphere the shear tensor vanishes, the rotation term, or angular momentum is not negligible. A simple approach preserving spherical symmetry is to assume that the particles are described by a random distribution of angular momenta such that the mean angular momentum at any point in space is zero [8]. Nevertheless, in any proper extension of the SSCM both effects need to be considered [9] since shear induces contraction while vorticity induces expansion as expected from a centrifugal effect.

In this letter, we study the net physical effect of shear and rotation in the framework of an extended spherical collapse model (ESCM). We restrict our analysis to the Einstein-de Sitter (EdS) and the flat  $\Lambda$ CDM background cosmologies. For the  $\Lambda$ CDM model we assume the following cosmological parameters:  $\Omega_m = 0.274$ ,  $\Omega_\Lambda = 0.726$  and  $h = 0.7$ . In particular, we discuss how the linear density threshold for collapsing non-relativistic component ( $\delta_c$ ) and its virial overdensity ( $\Delta_v$ ) change. We recall that the change of these two parameters has a strong effect on the mass function and other fundamental cosmological quantities. As a general result, it is also found

\*Electronic address: adelpopolo@astro.iag.usp.br

†Electronic address: francesco.pace@port.ac.uk

‡Electronic address: limajas@astro.iag.usp.br

that the extra terms appearing in the ESCM is responsible for higher values of the linear overdensity parameter at galactic scales as compared to the case without shear and rotation.

## II. DERIVATION OF $\delta_c$ AND $\Delta_V$

To begin with, let us now consider that the only clustering component in the cosmic medium is the cold dark matter. Following standard lines, the evolution of the overdensity  $\delta$  is driven by a second order non-linear differential equation [10, 11]:

$$\delta'' + \left( \frac{3}{a} + \frac{E'}{E} \right) \delta' - \frac{4}{3} \frac{\delta'^2}{1 + \delta} - \frac{3}{2} \frac{\Omega_m}{a^5 E^2(a)} \delta(1 + \delta) - \frac{1}{a^2 H^2(a)} (\sigma^2 - \omega^2)(1 + \delta) = 0, \quad (1)$$

where  $a(t)$  is the scale factor and the scalars,  $\sigma^2 = \sigma_{ij}\sigma^{ij}$  and  $\omega^2 = \omega_{ij}\omega^{ij}$ , denote the shear and rotation terms, respectively ( $\sigma_{ij}$  and  $\omega_{ij}$ , are the shear and vorticity tensors). The quantity  $\Omega_m$  is the present day value of the density parameter of the DM component while the quantity  $E(a)$  is defined by:

$$E(a) = \sqrt{\frac{\Omega_m}{a^3} + \Omega_\Lambda}, \quad (2)$$

where  $\Omega_\Lambda$  is the present day value of the vacuum density parameter (at  $a = 1$ ).

Following Pace et al. [11] we now calculate the threshold for the collapse,  $\delta_c$ , and the virial overdensity,  $\Delta_V$ . We look for an initial density contrast such that the non-linear equation diverges at the chosen collapse time. Once the initial overdensity is found, we use this value as an initial condition in the linearised version of Eq. 1.

The virial overdensity is readily obtained by using the definition  $\Delta_V = \log(\delta_{nl} + 1) = \zeta(x/y)^3$ , where  $x = a/a_{ta}$  is the normalized scale factor and  $y$  is the radius of the sphere normalized to its value at the turn-around.

In order to calculate the shear and vorticity terms in Eq. 1, we first recall that  $\delta = \rho/\bar{\rho} - 1 = (a/R)^3 - 1$ . As one may check, by inserting this expression into Eq. 1, the evolution equation for the density fluctuations  $\delta$  becomes [9, 12, 13]

$$\frac{d^2 R}{dt^2} = \frac{4}{3} \pi G \rho R - (\sigma^2 - \omega^2) \frac{R}{3} = -\frac{GM}{R^2} - (\sigma^2 - \omega^2) \frac{R}{3}, \quad (3)$$

which should be compared with the usual expression for the SCM with angular momentum (e.g., [14–16]):

$$\frac{d^2 R}{dt^2} = -\frac{GM}{R^2} + \frac{L^2}{M^2 R^3} = -\frac{GM}{R^2} + \frac{4}{25} \Omega^2 R, \quad (4)$$

where in the last expression we have replaced the momentum of inertia of a sphere,  $I = 2/5 MR^2$ . The previous

argument shows that vorticity,  $\omega$ , is strictly connected to angular velocity,  $\Omega$ .

In the simple case of a uniform rotation with angular velocity  $\Omega = \Omega_z \mathbf{e}_z$ , we have that  $\Omega = \omega/2$  (see also Chernin [17], for a more complex and complete treatment of the interrelation of vorticity and angular momentum in galaxies).

It is also convenient to define the dimensionless  $\alpha$ -number as the ratio between the rotational and the gravitational term:

$$\alpha = \frac{L^2}{M^3 R G}. \quad (5)$$

The above quoted ratio,  $\alpha$ , is of the order of 0.4, for a spiral galaxy like the Milky Way ( $L \simeq 2.5 \times 10^{74} g \text{ cm}^2/s$ ;  $R \simeq 15 \text{ kpc}$  [18, 19]), larger for smaller size perturbations (dwarf galaxies size perturbations) and smaller for larger size perturbations (for galaxy clusters the ratio is of the order of  $10^{-6}$ ).

Now, in order to integrate Eq. 1 one should determine how the extra term involving the difference  $\sigma^2 - \omega^2$  depends on the density contrast. Based on the above outlined argument for rotation one may calculate the same ratio between the gravitational and the extra term appearing in Eq. 1 thereby obtaining

$$(\sigma^2 - \omega^2) H_0^{-2} = -\frac{3}{2} \frac{\alpha \Omega_{m,0}}{a^3} \delta. \quad (6)$$

In the absence of a first principle workable expression, in what follows we will assume a more general power-law expression:

$$(\sigma^2 - \omega^2) H_0^{-2} \equiv -b \delta^n / a^3, \quad (7)$$

with the proviso that the constant parameters  $b$  and  $n$  can depend on the scale of the perturbations. In this concern, we remark that at the non-linear level, the gravitational term is  $(1 - \alpha)$  times smaller than for the case where rotation and shear is absent. This will have considerable effects on the linear and virial parameters of the spherical collapse model, as we will see in the next section. In particular, the values of  $b$  and  $n$  can be calculated comparing the threshold of collapse,  $\delta_c$ , in Sheth & Tormen [21] (later used to obtain the Sheth & Tormen [20] mass function) with the  $\delta_c$  parameter which is obtained from Eq. 1. In this way, we have obtained  $n = 1$  and  $b = 0.157$ , for galaxy size perturbations.

## III. BASIC RESULTS

In this section we discuss some physical consequences of the extended spherical collapse model discussed here. In particular, we obtain the linear overdensity parameter  $\delta_c$  and the virial overdensity  $\Delta_V$ .

In Figure 1 (four plots) we show the evolution of the overdensity  $\delta$  as a function of the scale factor assuming EdS and flat  $\Lambda$ CDM background cosmologies.

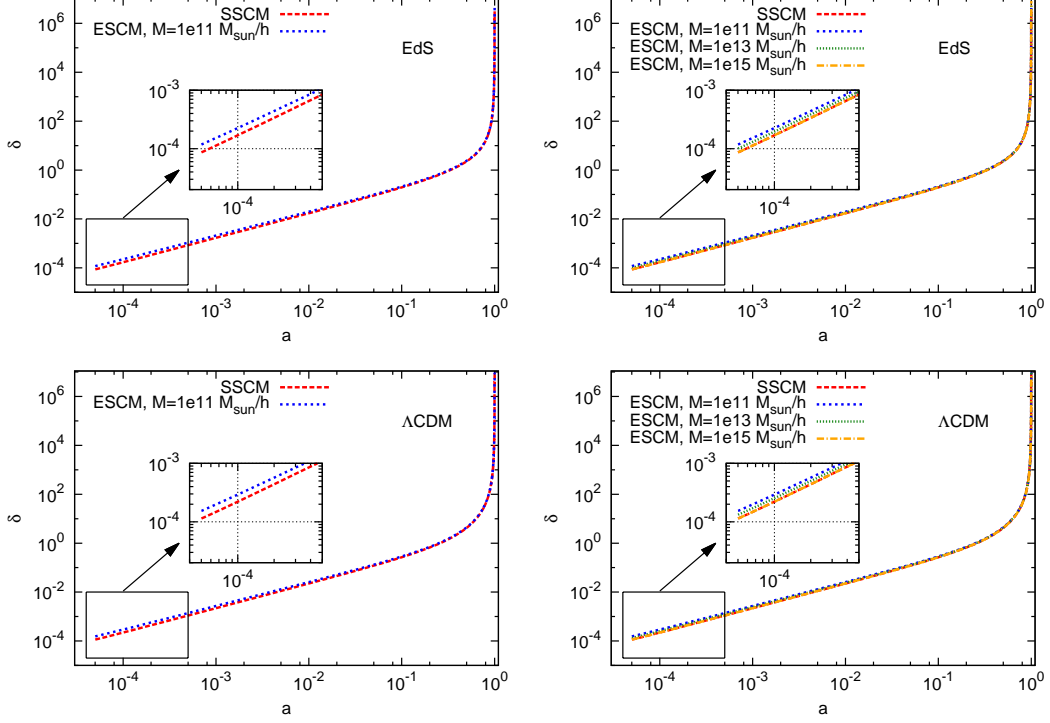


FIG. 1: The overdensity  $\delta$  as a function of the scale factor  $a$ . On the left panels we compare the evolution of the perturbation for the SSCM (red dashed curve) and ESCM (blue short-dashed) of  $10^{11} M_{\odot}/h$  in the framework of an EdS and flat  $\Lambda$ CDM cosmologies. On the right panels we show similar plots but now for objects with different mass scales. The blue dashed curve shows results for a  $10^{11} M_{\odot}/h$  object (galaxy), the green short-dashed curve a  $10^{13} M_{\odot}/h$  (galaxy group) while the orange dot-dashed curve represents a  $10^{15} M_{\odot}/h$  object (galaxy cluster). The images appearing the above plots display a zoom on the initial overdensity necessary for the collapsing halo at redshift  $z = 0$ . Note that the initial overdensity of the ESCM halos need to be higher regardless of the background cosmology.

In the left panels we compare the time evolution of  $\delta$  for the ESCM (short dashed blue curve) and the SSCM cases (red dashed line). The ESCM halo has a mass of  $10^{11} M_{\odot}/h$ . Since we want that both halos (spherical and non-spherical) should collapse at present time ( $a = 1$ ), the curves perfectly overlap in the non-linear regime. Therefore differences between the two considered models must take place at very early times, reflecting therefore in the different initial conditions. For instance, whether the collapse process is delayed in the case of ESCM halos (in comparison to the SCM description), one should expect that the initial overdensity must be higher in order to have the same efficiency. This is indeed the case, as we can see in the zoom panel on the left plot. In order to collapse at the same time of the non-rotating sphere, the initial overdensity has to be higher. For the non-rotating sphere we have  $\delta_i \approx 8.6 \times 10^{-5}$  while for the rotating sphere we have  $\delta_i \approx 1.2 \times 10^{-4}$ , with an increase therefore of approximately 28%.

It should be noticed that in the right panels we show how the overdensities evolve for different masses of the corresponding halos (SSCM and ESCM) for the same background cosmologies. As expected, the influence of any departure from spherical symmetry decreases with the increase of the mass. For all practical purposes, we

see that for scales of the order of  $10^{15} M_{\odot}/h$  the solutions of the SCM are recovered.

In Fig. 2 (4 plots), we show the evolution of the linear overdensity parameter  $\delta_c$  (upper panels) and of the virial overdensity  $\Delta_v$  (lower panels) for the same EdS and  $\Lambda$ CDM cosmologies. In the left panels, the analyses based on the ESCM are restricted to a halo of  $10^{11} M_{\odot}/h$  since for galactic masses the effect will be enhanced, while on the right panels we consider also the effect of distinct masses. As before, we concentrate our analyses to three different mass scales: galactic ( $\approx 10^{11} M_{\odot}/h$ ), groups ( $\approx 10^{13} M_{\odot}/h$ ) and clusters ( $\approx 10^{15} M_{\odot}/h$ ). As expected from the analysis of Fig. 1, with the growth of the mass the effect of the extra term in the ESCM becomes negligible, and we recover the same values of the SSCM case. It is also worth to notice that the results for the  $\Lambda$ CDM model reduce to the ones of the EdS model for sufficiently high redshifts, since the influence of the cosmological constant becomes rapidly negligible. We will therefore concentrate only on the analysis of the left panels. For the different line colours and styles, we remind to the caption of the figure.

As expected, the  $\delta_c$  for the ESCM is  $\approx 40\%$  higher than for the SSCM case and it decreases towards high redshifts, since the effect of the extra term becomes smaller.

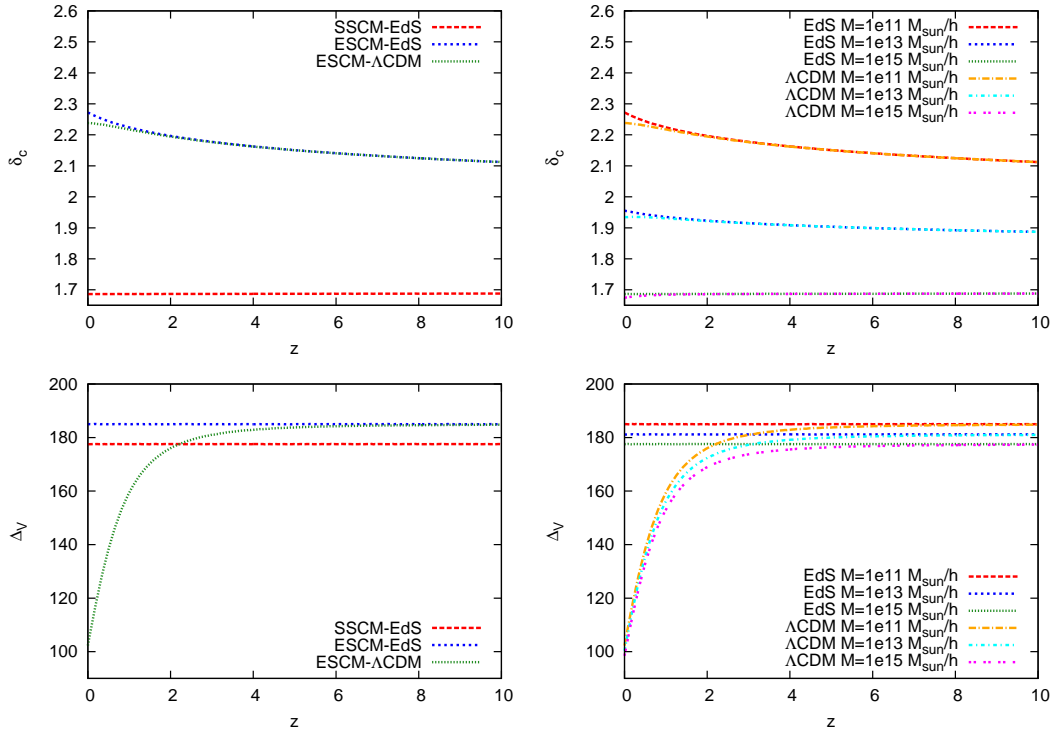


FIG. 2: Upper (lower) panels: evolution of  $\delta_c$  ( $\Delta_V$ ) with respect to the redshift  $z$  for the EdS and the  $\Lambda$ CDM models. The left panels show the time evolution for both parameters at galactic scale ( $10^{11} M_\odot/h$ ). The red curve represents the solution for the EdS model in the non-rotating case while the blue and the green curves represent the EdS and the  $\Lambda$ CDM model when rotation is included. On the right panels we compare the time evolution for three different masses,  $10^{11}$ ,  $10^{13}$  and  $10^{15} M_\odot/h$ . Different colours and line-styles correspond to different masses and different cosmological models: red dashed (orange dot-dashed) curve represents a halo of  $10^{11} M_\odot/h$  in a EdS ( $\Lambda$ CDM) cosmological background, blue short-dashed (dot-short-dashed cyan) curve represents a halo of  $10^{13} M_\odot/h$  for an EdS ( $\Lambda$ CDM) model, while the green dotted (magenta) curve stands for an object of  $10^{15} M_\odot/h$  in an EdS ( $\Lambda$ CDM) model.

For the EdS model,  $\delta_c$  decreases from a value of  $\approx 2.3$  at  $z = 0$  to  $\approx 2.1$  at  $z = 10$ . As expected, the linear overdensity parameter for the  $\Lambda$ CDM model is smaller than the EdS one. This is understood by taking into account that if we want to have the same number of structures now, we need to have a faster growth of structures to overcome the influence of the cosmological constant. This translates into a lower  $\delta_c$ .

In the lower panels we compare the behaviour of  $\Delta_V$  in the SSCM approach with the one predicted by the ESCM description. The red dashed (blue short dashed) curve show the standard and the extended results for an EdS model while the green dotted curve represents a  $\Lambda$ CDM model. It is clear that the ESCM description affects also the virial overdensity parameter. In particular, we see that  $\Delta_V$  is always constant in time for the EdS model. However, with the extra term its value increases reaching  $\Delta_V \approx 185$ , about 4% higher than the standard result. The curve for the  $\Lambda$ CDM model approximates the EdS at high redshifts, as expected. Once again higher masses are less affected by the ESCM correction term (lower right panel).

#### IV. CONCLUSIONS

In this letter we have discussed how shear and rotation affect the standard spherical collapse model. The net effect of such quantities which is  $\propto (\sigma^2 - \omega^2)$  has been phenomenologically described by a power law on the density contrast depending on two parameters ( $b$  and  $n$ ). It was also shown that the values of  $b$  and  $n$  can be calculated by comparing the threshold of collapse,  $\delta_c$ , as discussed in Sheth & Tormen [21], with the  $\delta_c$  value which is directly obtained from Eq. 1. We have focused our discussion on the influence of such an extra term on the spherical collapse parameters  $\delta_c$  and  $\Delta_V$ . As it should be expected, the extra term slows down the collapse, and, as such, higher values for the initial perturbations are required in order to have a collapse at the same time of a spherical collapsing sphere. It is also found that the extra term contribution is more important for galactic scales so that its contribution becomes negligible at high masses (galaxy clusters). This is shown explicitly in Figure 1.

Finally, in Figure 2 we have numerically evaluated and compared the evolutionary behaviour of both the ESCM and SSCM approaches. We have seen that both the lin-

ear and the non-linear virial overdensity in the extended spherical collapse model are enhanced with respect to the standard spherical case. Enhancements are more pronounced for  $\delta_c$  ( $\approx 40\%$ ), while for  $\Delta_V$  are only of the order of few percent.

These results reinforce the importance of a more complete and rigorous treatment involving the effects of shear and rotation at the late stages of the collapsing halo history mainly for the galactic scales. A more detailed article including the calculations of the cumulative mass

function will be published elsewhere.

**Acknowledgements:** ADP is partially supported by a visiting research fellowship from FAPESP (grant 2011/20688-1), and wishes also to thank the Astronomy Department of São Paulo University for the facilities and hospitality. FP is supported by STFC grant ST/H002774/1, and JASL is also partially supported by CNPq and FAPESP under grants 304792/2003-9 and 04/13668-0.

- 
- [1] A. Riess *et al.*, *Astron. J.* **116**, 1009 (1998); S. Perlmutter *et al.*, *Nature* **391**, 51 (1998); M. Kowalski *et al.*, *Astrophys. J.* **686**, 749 (2008); R. Amanullah *et al.*, *Astrophys. J.* **716**, 712 (2010).
  - [2] D. N. Spergel *et al.*, *Astrophys. J. Suppl.* **148**, 175 (2003); D. N. Spergel *et al.*, *Astrophys. J. Suppl. Ser.* **170**, 377 (2007).
  - [3] S. W. Allen *et al.*, *Mon. Not. R. Astr. Soc.* **334**, L11 (2002); J. A. S. Lima, J. V. Cunha and J. S. Alcaniz, *Phys. Rev. D* **68**, 023510 (2003), [astro-ph/0303388](#); D. J. Eisenstein *et al.*, *Astrophys. J.* **633**, 560 (2005); S. W. Allen *et al.*, *Mon. Not. Roy. Ast. Soc.* **383**, 879 (2008).
  - [4] T. Padmanabhan, *Phys. Rep.* **380**, 235 (2003); P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003); J. A. S. Lima, *Braz. J. Phys.* **34** (2004), [[astro-ph/0402109](#)]; J. A. Frieman, M. S. Turner and D. Huterer, *Ann. Rev. Astron. and Astrophys.* **46**, 385 (2008); M. Li *et al.*, [arXiv:1103.5870](#) (2011).
  - [5] E. Komatsu *et al.*, *Astrophys. J.* **192**, 18 (2011); S. W. Allen, A. E. Evrard and A. B. Mantz, *Ann. Rev. Astron. Astrophys.* **49**, 409 (2011).
  - [6] Gunn J. E. and Gott J. R., *Astrophys. J.* **176**, 1 (1972)
  - [7] B. S. Ryden and J. E. Gunn, *Astrophys. J.* **318**, 15 (1987); A. V. Gurevich and K. P. Zybin, *Zhurnal Eksperimental noi i Teoreticheskoi Fiziki* **94**, 3 (1988); A. V. Gurevich and K. P. Zybin, *Zhurnal Eksperimental noi i Teoreticheskoi Fiziki* **94**, 5 (1988); S. D. M. White and D. Zaritsky, *Astrophys. J.* **394**, 1 (1992); P. Sikivie, I. I. Tkachev and Y. Wang, *Phys. Rev. D.* **56**, 1863 (1997); M. Le Delliou and R. N. Henriksen, *Astron. Astrophys* **408**, 27 (2003); L. L. R. Williams, A. Babul and J. J. Dalcanton, *Astrophys. J.* **604**, 18 (2004); S. Basilakos, M. Plionis and J. Solà, *Phys. Rev. D* **82**, 083512 (2010).
  - [8] S. D. M. White and D. Zaritsky, *Astrophys. J.* **394** 1, (1992).
  - [9] S. Engineer, N. Kanekar, T. Padmanabhan, *Mon. Not. R. Astron. Soc.* **314**, 279 (2000).
  - [10] T. Padmanabhan, *Structure Formation in The Universe*, Cambridge University Press, Cambridge, England (1993); Y. Ohta, I. Kayo, A. Taruya, *Astrophys. J.* **589**, 1 (2003).
  - [11] F. Pace, J. C. Waizmann and M. Bartelmann, *Mon. Not. R. Ast. Soc.* **406**, 1865 (2010).
  - [12] P. Fosalba, E. Gaztanaga and F. J. Castander, 2003, *Astrophys. J. Let.* **597**, L89 (2003).
  - [13] Y. Ohta, I. Kayo and A. Taruya, 2003, *Astrophys. J.* **589**, 1 (2003).
  - [14] P. J. E. Peebles, *Principles of Physical Cosmology*, Princeton University Press (1993).
  - [15] A. Nusser, *Mon. Not. R. Astron. Soc.* **325**, 1397 (2001).
  - [16] P. Zukin and E. Bertschinger, *Phys. Rev. D* **82**, 104044 (2010).
  - [17] A. D. Chernin, *Astron. Astrophys.* **267**, 315 (1993).
  - [18] B. S. Ryden and J. E. Gunn, *Astrophys. J.* **318**, 15 (1987).
  - [19] P. Catelan and T. Theuns, *Mon. Not. R. Astron. Soc.* **282**, 436 (1996).
  - [20] R. K. Sheth and G. Tormen, *Mon. Not. R. Astron. Soc.* **323**, 1 (2001). See also, A. D. Popolo, *Astron. and Astrophys.* **344**, 17 (1999); *ibidem* **357**, 809 (2000).
  - [21] R. K. Sheth and G. Tormen, *Mon. Not. R. Astron. Soc.* **329**, 61 (2002).